FILTRATION OF LIQUID AND VAPOR THROUGH A POROUS WALL

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A relation is derived for determining the mass flow rate of coolant, depending on the location of the phase-transition zone and as a function of the physical properties of the system.

The feasibility of transpiration cooling with a liquid which changes phase inside the wall has recently received much attention from researchers [1, 2], on account of the obvious advantages over such cooling with a gas. It is not possible in practice, however, to ensure a constant flow rate of coolant when the phase-transition zone [1] or the beginning of it [2] moves, because then the hydraulic drag of the porous plate changes. The phenomenon of the drag increasing during phase transition, quite well known in the case of the flow of boiling water through pipes [3], has been discovered only recently [4] in the case of filtration through porous bodies. Producing the necessary pressure drop appears a logical and the only possible condition for establishing the flow rate of coolant through a porous wall.

The problem is depicted schematically in Fig. 1. A constant pressure drop $p_0 - p_1 > 0$ drives a liquid through a flat homogeneous plate whose y and z dimensions are much larger than its thickness δ. The state of the liquid changes while the latter flows through this plate: it evaporates within a thin layer $x = L$, if the outer thermal flux density q is sufficiently high. The problem is to determine how the coolant flow rate varies depending on the location of the evaporation zone inside the plate. The steady-state one-dimensional flow of liquid through a porous medium is described by the semiempirical equation [5]:

$$-\frac{dp}{dx} = \frac{\alpha\mu G}{\rho} + \frac{\beta}{\rho} G^2, \tag{1}$$

where G denotes the mass flow rate of filtrating liquid and α, β denote the coefficients of viscous and inertial drag, respectively. The second term on the right-hand side represents the departure from Darcy's law at higher velocities, this departure being due to inertia effects in a porous body, i.e., in a multitude of curvilinear channels with variable cross sections.

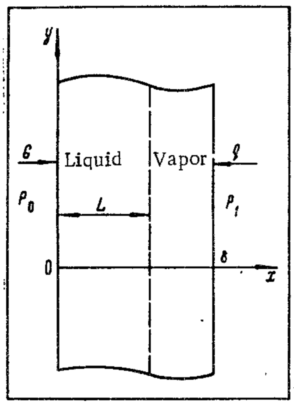

Fig. 1. Physical diagram of the problem.

The continuity equation for steady flow

$$\frac{dG}{dx} = 0 \tag{2}$$

indicates a constant flow rate at any cross section:

$$G(x) = \text{const.} \tag{3}$$

Integrating expression (1) separately for the liquid zone and the vapor zone over the entire plate, we obtain

$$p_0 - p_1 = \alpha G \left[\int_0^L \frac{\mu(x)}{\rho(x)} dx + \int_L^\delta \frac{\mu(x)}{\rho(x)} dx \right] + \beta G^2 \left[\int_0^L \frac{dx}{\rho(x)} + \int_L^\delta \frac{dx}{\rho(x)} \right]. \tag{4}$$

The values of $\mu(x)$ and $\rho(x)$ for both zones can be found in tables, if the pressure and the temperature are known at every point in the system, i.e., if the

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TABLE 1. Physical Properties of Water and of Water Vapor along the Saturation Curve

p_{sat}, bar	T_{sat}, °C	$\frac{\nu_2}{\nu_1}$	$\frac{\rho_1}{\rho_2}$
0,1	6,98	74,5	$1{,}29\cdot10^5$
1	99,63	73,5	$1{,}69\cdot10^3$
10	179,88	17,5	$1{,}76\cdot10^2$
100	310,96	2,94	12,8

problem of convective heat transfer has been solved. In our case we will assume, to the first approximation, that the physical properties of both liquid and vapor are constant and for their values we will take those corresponding to the saturation line at a pressure $p_{sat} = (p_0 + p_1)/2$. Denoting the dynamic viscosity and the density of the liquid respectively by $\mu(x) = \mu_1$ and $\rho(x) = \rho_1$ for $x \in [0, L]$, and considering them constant, we rewrite (4) first as

$$p_0 - p_1 = \alpha G\left[\frac{\mu_1}{\rho_1}L + \frac{\mu_2}{\rho_2}(\delta - L)\right] + \beta G^2\left[\frac{L}{\rho_1} + \frac{(\delta - L)}{\rho_2}\right], \qquad (5)$$

and then, after dividing both sides by δ and $\mu_1/\rho_1 = \nu_1$, in a form more convenient for analysis:

$$\frac{p_0 - p_1}{\delta\nu_1} = \alpha G\left[l + \frac{\nu_2}{\nu_1}(1 - l)\right] + \frac{\beta G^2}{\mu_1}\left[l + \frac{\rho_1}{\rho_2}(1 - l)\right], \qquad (6)$$

with l denoting the dimensionless coordinate of the phase-transition zone. If we let $l = 1/2$, then it is possible to easily identify the contribution by each of the equally long zones to the total hydraulic drag:

$$\frac{p_0 - p_1}{\delta\nu_1} = \frac{\alpha G}{2}\left(1 + \frac{\nu_2}{\nu_1}\right) + \frac{\beta G^2}{2\mu_1}\left(1 + \frac{\rho_1}{\rho_2}\right). \qquad (7)$$

The second terms in the parentheses, namely, the ratio of liquid and vapor kinematic viscosities and the ratio of liquid and vapor densities, represent, respectively, the viscous and the inertial drag of the zone where vapor flows, relative to the total respective drag of the plate at a constant mass flow rate of coolant. Their order of magnitude can be estimated from the tables of physical properties along the saturation line for water and water vapor [6].

These data point toward the predominant role of vapor in the total drag. As the pressure rises, the difference between the phases fades until it becomes negligible near the critical point.

We transform Eq. (6) further, namely, divide both sides by α and introduce, for simplicity, the following symbols:

$$m = \left[l + \frac{\nu_2}{\nu_1}(1 - l)\right], \quad n = \left[l + \frac{\rho_1}{\rho_2}(1 - l)\right], \quad G_1 = \frac{p_0 - p_1}{\delta\nu_1\alpha}. \qquad (8)$$

Here G_1 signifies physically the mass flow rate of a liquid according to Darcy. We then obtain

$$G_1 = Gm + \frac{G^2}{\mu_1}\cdot\frac{\beta}{\alpha}n. \qquad (9)$$

Finally, Eq. (9) is conveniently converted into a dimensionless one with the relative flow rate $g = G/G_1$ and the Reynolds number $Re = G_1/\mu_1 \cdot \beta/\alpha$:

$$1 = gm + g^2 n\,Re. \qquad (10)$$

The Reynolds number is defined here as the ratio of the inertial drag force to the viscous drag force, with the respective coefficients in the equation yielding the characteristic dimension. The solution of Eq. (10) depends on l, ν_2/ν_1, ρ_1/ρ_2 and $Re = (p_0 - p_1)/\delta\nu_1\alpha\mu_1 \cdot \beta/\alpha$. Parameters ν_2/ν_1, ρ_2/ρ_1, μ_1, and ν_1 are not independent in our formulation of the problem. They are all uniquely related to the saturation pressure.

As an example, let us consider the filtration of water through a porous plate of stainless steel and a $\delta = 5$ mm thick with a 30% porosity and the following drag coefficients [5, 7]: viscous drag $\alpha = 3.5\cdot10^{12}$ m^2 and inertial drag $\beta = 1.2\cdot10^{-7}$ m^{-1}. The solution to Eq. (10) for the saturation states, according to Table 1, is shown in Fig. 2a, in terms of the mass flow rate as a function of the location of the phase-transition zone inside the plate. The Reynolds number $Re = 0.1$ corresponds to the following thermal flux density outside:

$$q \approx G_1 r = Re\,\mu_1\frac{\alpha}{\beta}r \approx Re\cdot10^8 \approx 10^7\ \mathrm{W\cdot m^{-2}}$$

The most important feature of the curves is the rapid decrease in the coolant flow rate when the boiling process shifts from the surface deeper into the plate. When the outer thermal flux and the pressure drop

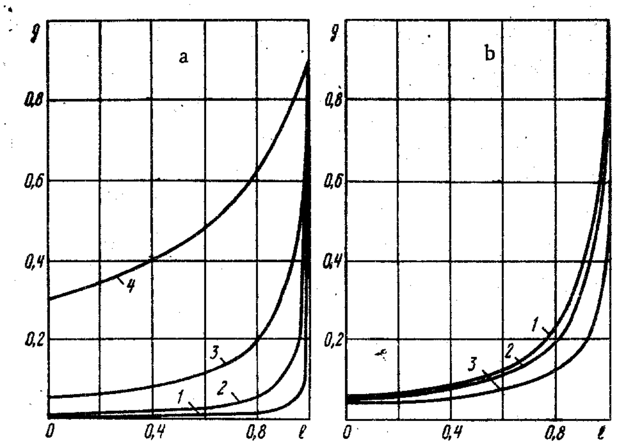

Fig. 2. Dimensionless mass flow rate of coolant, as a function of the location of the phase-transition zone: a) for Re = 0.1 [1) p_{sat} = 0.1 bar; 2) 1.0 bar; 3) 10 bar; 4) 100 bar]; b) at p_{sat} = 10 bar [1) Re = 0; 2) Re = 0.1; 3) Re = 1.0].

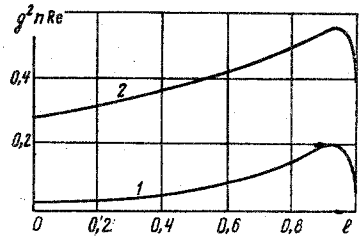

Fig. 3. Inertial component of drag, as a function of the location of the phase-transition zone, at p_{sat} = 10 bar: 1) Re = 0.1; 2) Re = 1.0.

across the plate remain constant, then the phase-transition zone can continue to penetrate until it clears the wall.

A pressure rise in the system results in flatter curves with less variation in the flow rate. Such an effect is achieved by an increase in the Reynolds number at constant properties of the coolant, as shown in Fig. 2b.

The contribution by the inertial component to the total drag (second term on the right-hand side of expression (10)) is indicated in Fig. 3. The magnitude of the inertial drag does not appear constant and must be taken into account already when Re = 0.1.

NOTATION

G	is the dimensional mass flow rate of coolant;
g	is the dimensionless mass flow rate of coolant;
δ	is the thickness of a porous plate;
L	is the dimensional coordinate of the phase-transition zone;
l	is the dimensionless coordinate of the phase-transition zone;
x, y, and z	are the longitudinal coordinates;
q	is the outer thermal flux density;
p_0 and p_1	are the pressures at the plate surfaces;
α	is the viscous drag coefficient;
β	is the inertial drag coefficient;
μ	is the dynamic viscosity;
ν	is the kinematic viscosity;
ρ	is the density;
p_{sat}	is the saturation pressure;
Re	is the Reynolds number;
m and n	are the dimensionless quantities defined by relation (8).

Subscripts

1 and 2 denote the physical properties of liquid and vapor coolant, respectively.

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